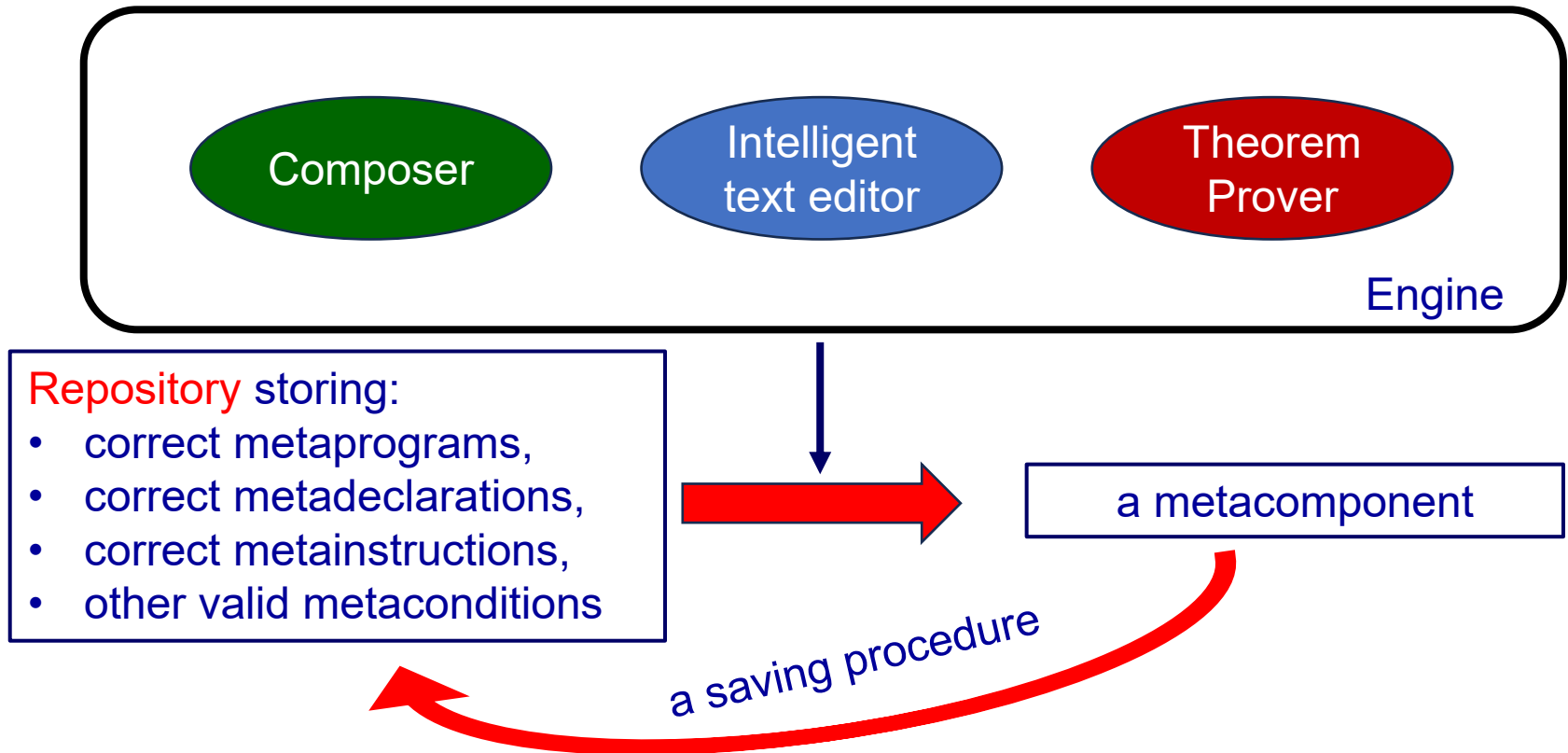


Investigations on ecosystems for Lingua-V programmers

Andrzej Jacek Blikle

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An ecosystem of programmers in Lingua-V



Steps of program-development regarded as proofs:

ex-ante constructive (implicit) proofs – involving Composer

ex-post analytic (explicit) proofs – involving Theorem Prover

We need a formalized theory where such proofs may be carried out.

Our way to a formalized theory of **Lingua-V**

1. Extending the denotational model of **Lingua** to **Lingua-V**:
 - a. extending **AlgSyn** to **AlgSyn-V**,
 - b. extending **AlgDen** to **AlgDen-V**.
2. Lifting a denotational model of **Lingua-V** to **Lingua-FT**:
 - a. lifting **AlgSyn-V** to **AlgSyn-FT**,
 - b. lifting **AlgDen-V** to **AlgDen-FT**.
3. Choosing an axiomatic framework for **Lingua-FT**:
 - a. extending **Lingua-FT** to **Lingua-AFT** (axiomatic FT),
 - b. establishing a set of axioms for which **AlgDen-FT** is a model.
4. Building a denotational model of **Lingua-E** – a language of an ecosystem:
 - a. Building an algebra of denotations **AlgDen-E**
 - b. Building an algebra of syntax **AlgSyn-E**

A hierarchy of languages:

Lingua	– a source programming language,
Lingua-V	= Lingua + conditions + metaconditions
Lingua-FT	= Lingua-V + variables
Lingua-AFT	= Lingua-FT + axioms-oriented-elements
Lingua-E	– Lingua-AFT -based language of ecosystem

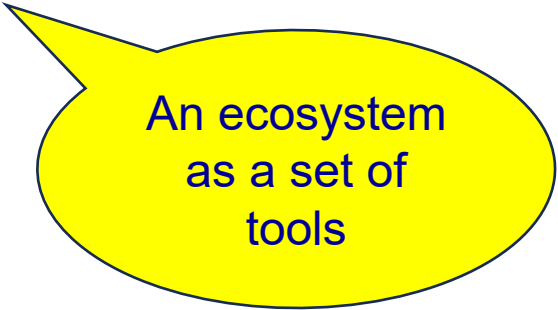
running over
denotations of
Lingua-V

The need of a formalized theory

We need a formalized theory rich enough to prove
lemmas about metacomponents
and in particular
the soundness of program-construction rules

A formalized theory:

- a formal language **Lingua-AFT**,
- a set of axioms,
- a set of inference rules.



An ecosystem
as a set of
tools

A recollection of formalized theories (1)

First-order theories – preliminaries

In first-order theories we talk about:

ele : Uni — **elements** of a set called a **universe**
fun : Uni^{cn} $\mapsto$ Uni — **functions** with $n \geq 0$
pre : Uni^{cn} $\mapsto$ Bool — **predicates** with $n \geq 0$

A language of first-order theories includes two syntactic categories

terms — represent functions
formulas — represent predicates

names and separators
will be printed in green

variables will be printed
in black

Primitives of syntax

var : Variable — **variables** (running over Uni)
fn : Fn — function names
pn : Pn — predicate names
sep : Separator — separators, e.g.: „ (” , „) ” , „ , ...

Alphabet = Variable | Fn | Pn | Separator

arity : Fn | Pn $\mapsto$ {0, 1, 2,...} — **arity** of names

A recollection of formalized theories (2)

First-order theories – the language

var : Variable =

x | y | z | x-1 | y-1 | z-1 | ... variables may have indices

ter : Term =

mk-term(Variable)		make a term from a variable
fn()		for all fn : Fn with arity.fn = 0
fn(<u>Term, ... ,Term</u>)		for all fn : Fn with arity.fn = n

for : Formulaⁿ =

true		
false		
pn(<u>Term, ... ,Term</u>)		for all pn : Pn with arity.pn = n

not(Formula ⁿ)	
and(Formula, Formula)	
or(Formula, Formula)	
implies(Formula, Formula)	
($\forall$ Variable) Formula	
($\exists$ Variable) Formula	

ground formulas	– no variables: $1 < 2$
free formulas	– with variables: $x < x+1$

A recollection of formalized theories (3)

Interpretation and semantics (1)

An **interpretation** of a language of a formalized theory:

$\text{Int} = (\text{Uni}, F, P)$

with

Uni – set called **universe**, its elements are called **primitive elements**,

F – function; $F[\text{fn}] : \text{Uni}^{\text{cn}} \mapsto \text{Uni}$ for $\text{arity.fn} = n$
 $F[\text{fn}] : \mapsto \text{Uni}$ for $\text{arity.fn} = 0$

P – function; $P[\text{pn}] : \text{Uni}^{\text{cn}} \mapsto \text{Bool}$;
 $P[\text{true}] = \text{tt}$, $P[\text{false}] = \text{ff}$

A **valuation** is a total function that assigns elements of Uni to variables:

$\text{vlu} : \text{Valuation} = \text{Variable} \mapsto \text{Uni}$

The **semantics** of variables, terms and formulas:

$\text{SV} : \text{Variable} \mapsto \text{Variable}$

$\text{ST} : \text{Term} \mapsto \text{Valuation} \mapsto \text{Uni}$

$\text{SF} : \text{Formula} \mapsto \text{Valuation} \mapsto \text{Bool}$

A recollection of formalized theories (4)

Interpretation and semantics (2)

The **semantics of variables**:

$ST : \text{Variable} \mapsto \text{Variable}$

$ST.[\text{var}] = \text{var}$

The **semantics of terms**:

$ST : \text{Term} \mapsto \text{Valuation} \mapsto \text{Uni}$

$ST.[\text{mk-term}(\text{var})].\text{vlu} = \text{vlu}.\text{var}, \quad \text{var} : \text{Variable}$

$ST[\text{fn}(\text{ter-1}, \dots, \text{ter-n})].\text{vlu} = F[\text{fn}].(ST[\text{ter-1}].\text{vlu}, \dots, ST[\text{ter-n}].\text{vlu}) \quad \text{arity}.\text{fn} = n$

The **semantics of formulas**:

$SF : \text{Formula} \mapsto \text{Valuation} \mapsto \text{Bool}$

$SF[\text{true}].\text{vlu} = \text{tt}$

$SF[\text{false}].\text{vlu} = \text{ff} \quad \text{and, not are classical metaoperations}$

$SF[\text{pn}(\text{ter-1}, \dots, \text{ter-n})].\text{vlu} = P[\text{pn}].(ST[\text{ter-1}].\text{vlu}, \dots, ST[\text{ter-n}].\text{vlu}), \quad \text{arity}.\text{fn} = n$

$SF[(\text{for-1 and for-2})].\text{vlu} = SF[\text{for-1}].\text{vlu} \text{ and } SF[\text{for-2}].\text{vlu}$

$SF[\text{not}(\text{for})].\text{vlu} = \text{not } SF[\text{for}]$

$SF[(\forall \text{ var})\text{for}].\text{vlu} = \text{tt iff for every } \text{ele} : \text{Uni}, \quad \text{for.vlu}[\text{var/ele}] = \text{tt}$

$SF[(\exists \text{ var})\text{for}].\text{vlu} = \text{tt iff there exists } \text{ele} : \text{Uni}, \text{ such that } \text{for.vlu}[\text{var/ele}] = \text{tt}$

A recollection of formalized theories (5)

Satisfaction, models and validity

For a given interpretation:

$\text{Int} = (\text{Uni}, F, P)$

A formula for is said to be **satisfied** in Int if:

$\text{SF}[\text{for}].\text{vlu} = \text{tt}$ for every $\text{vlu} : \text{Valuation}$

An interpretation Int is said to be a **model** of a theory with set of axioms A if all axioms are satisfied in Int .

A formula for is said to be **valid** in a theory with a set of axioms A , in symbols

$A \vdash \text{for}$

if it is satisfied in every model of this theory.

E.g.: $\text{Peano} \vdash \text{zer} \neq \text{suc}(\text{zer})$

A recollection of formalized theories (6)

Deduction – a way of proving the validity of formulas

$A \models \text{for}$ for is a **theorem** in the theory with axioms A if it can be derived from A by means of **deduction rules**

The main deduction rules

Rule of substitution

$$\begin{array}{c} \downarrow A \models \text{for}(x) \\ \hline \downarrow A \models \text{for}(\text{ter}) \\ x \text{ free in } \text{for}(x) \\ \text{ter} - \text{an arbitrary term} \end{array}$$

Rule of generalization

$$\begin{array}{c} \uparrow A \models \text{for}(x) \\ \hline \downarrow A \models (\forall x) \text{for}(x) \\ x \text{ free in } \text{for}(x) \end{array}$$

Rule of detachment

$$\begin{array}{c} \downarrow A \models \text{for}_1 \\ \hline \downarrow A \models \text{for}_1 \rightarrow \text{for}_2 \\ \hline \downarrow A \models \text{for}_2 \end{array}$$

In **second-order theories** we have also

- functional variables,
- predication variables.

Gödel's completeness theorem

In every first-order theory with axioms A

$A \models \text{for}$ iff $A \models \text{for}$

Gödel's adequacy theorem

In every second-order theory with axioms A

if $A \models \text{for}$ then $A \models \text{for}$

Consistency and completeness of formalized theories

Definition

A theory is **consistent** if it has a model. niesprzeczna, spójna

Theorems:

1. A theory is consistent iff there is no formula ϕ such that ϕ and $\neg(\phi)$ are valid.
2. A theory is consistent iff there exists a formula ϕ for which ϕ is not valid.

Inconsistent theories do not distinguish between truth and falsity!

Definition pełna

A theory is **complete** if it is consistent and for any formula ϕ either $A \models \phi$ or $A \models \neg(\phi)$.

The majority of interesting theories are incomplete!

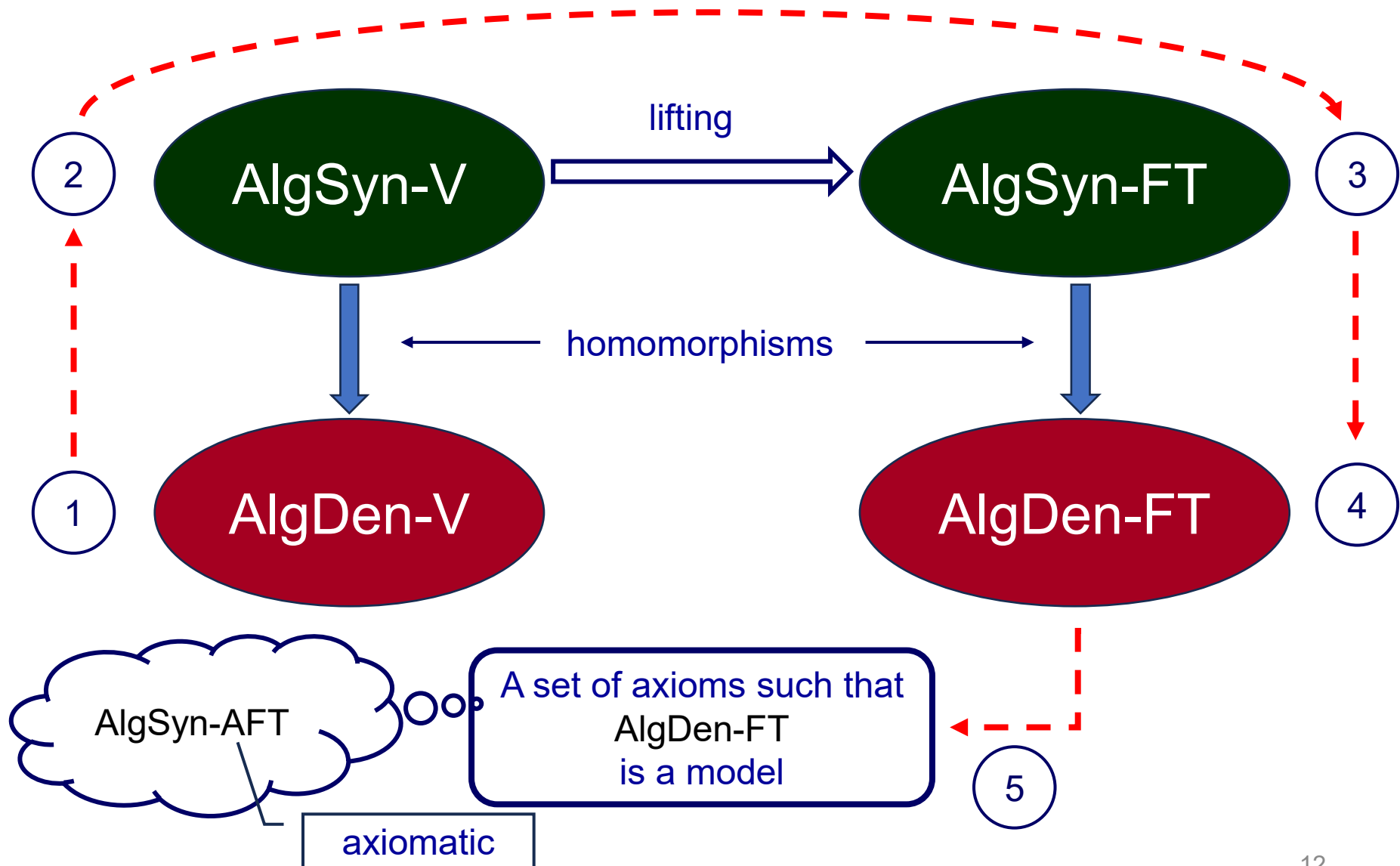
Observation

In every theory for any formula ϕ $A \models \phi$ (for **or** $A \models \neg(\phi)$).

Our theory of denotations must be consistent but will not be complete

Lifting Language-V to Language-FT (1)

FT – formal theory



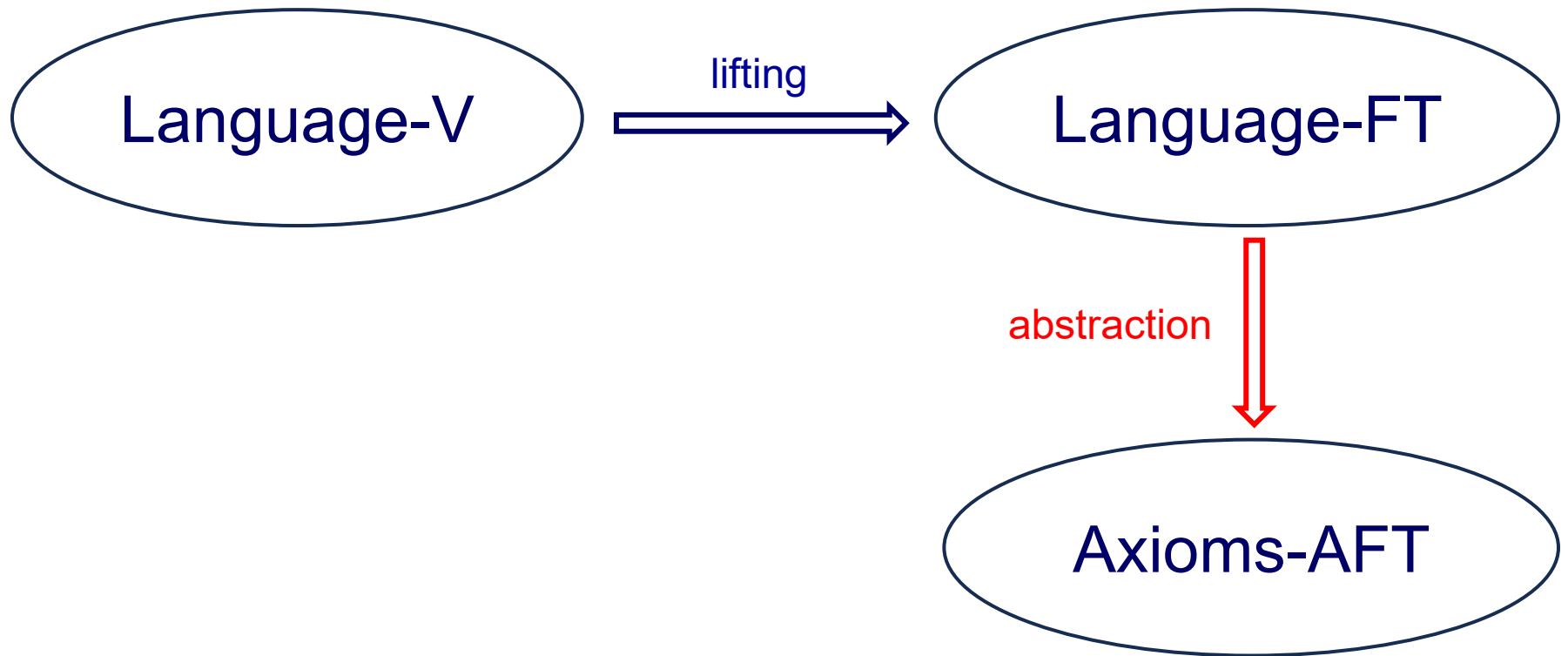
Lifting Language-V to Language-FT (2)

How theories and models are built

mathematical logic: **axioms** → **its models**

working mathematician: **a model** → **its axioms**

our case



Formalizing Lingua-V (1)

syntax – new categories

To the grammar of **Lingua**, we add the following categories:

con	: Con-V	— conditions
asr	: Asr-V	— assertions
sin	: SpeIns-V	— specified instructions
sde	: SpeDec-V	— specified declarations
sct	: SpeClaTra-V	— specified class transformations
spp	: SpeProPre-V	— specified program preambles
spr	: SpePro-V	— specified programs
mco	: MetCon-V	— metaconditions

Lifting Lingua-V to Lingua-FT (1)

syntax - variables

Individual variables – for all **cn** : Cn-V

inv : IdeVar-FT	= <u>ide</u>	<u>ide-1</u>	...	variables corresponding to identifies,
inv : ValExpVar-FT	= <u>vex</u>	<u>vex-1</u>	...	variables corresponding to val. expressions,
inv : RefExpVar-FT	= <u>rex</u>	<u>rex-1</u>	...	variables corresponding to ref.-expressions,
inv : SpeInsVar-FT	= <u>sin</u>	<u>sin-1</u>	...	variables corresponding to specinstructions,
inv : ConVar-FT	= <u>con</u>	<u>con-1</u>	...	variables corresponding to conditions,
...				
inv : MetConVal-FT	= <u>mec</u>	<u>mec-1</u>	...	variables corresponding to metaconditions.

Second-order variables:
vacate (so far)

Variables if **Lingua-FT** run over the
denotations of **Lingua-V**.

Lifting Lingua-V to Lingua-FT (2)

syntax – terms (1)

value expressions

vex : ValExp-FT =

vex-make-vex(ValExpVar-FT)		single-variable term
vex-bo(BooleanSyn-FT)		
vex-in(IntegerSyn-FT)		
vex-re(RealSyn-FT)		
vex-te(TextSyn-FT)		
vex-variable(Ide-FT)		single-identifier value-expression
vex-attribute(ValExp-FT , Ide-FT)		
vex-call-fun-pro(Ide-FT, Ide-FT, ActPar-FT)		
vex-add-int(ValExp-FT , ValExp-FT)		
vex-less-int(ValExp-FT , ValExp-FT)		
vex-or-m(ValExp-FT , ValExp-FT)		McCarthy's alternative
vex-create-li(ValExp-FT)		
vex-get-from-rc(ValExp-FT , Ide-FT)		

...

Note the difference:

vex – a metavariable running over domain ValExp-FT

vex – an individual variable of sort „value expression” in **Lingua-FT**

Lifting Lingua-V to Lingua-FT (3)

syntax - terms (2)

specinstructions

sin : SpeIns-FT =

sin-make-sin(SpeInsVar-FT)	single-variable term
sin-make-asr(Con-FT)	assertions
sin-skip-ins()	
sin-assign(RefExp-FT , ValExp-FT)	
sin-call-imp-pro(Ide-FT , Ide-FT , ActPar-FT , ActPar-FT)	
sin-call-obj-con(Ide-FT , Ide-FT , ActPar-FT)	
sin-if(ValExp-FT , SpeIns-FT , SpeIns-FT)	
sin-if-error(ValExp-FT , SpeIns-FT)	
sin-while(ValExp-FT , SpeIns-FT)	
sin-compose-ins(Ins-FT , Ins-FT)	

identifiers

ide : Ide-FT =

IdeVar-FT |
Identifier

Lifting Lingua-V to Lingua-FT (4)

syntax – terms (3)

conditions

con : Con-FT =

con-or-k(Con-FT , Con-FT)	Kleene's alternative
con-less-int(ValExp-FT , ValExp-FT)	
con-is-value(ValExp-FT)	
con-is-free(Ide-FT)	
con-left-algorithmic(SpeIns-FT , Con-FT)	
con-right-algorithmic(Con-FT , SpePro-FT)	

...

examples of ground terms (in abstract syntax)

sin-assign(x, vex-divide-re(1, z)) $x := 1/z$
vex-less(y, 0)
sin-while(vex-less-int(x, 0), sin-assign(x, vex-add-int(a, 1)))
sin-skip-ins

examples of free terms (note: symbols of variables are underlined)

sin-assign(rex, vex-divide-re(vex-1, vex-2))
vex-less(vex, 0)
sin-while(vex-less-int(vex-1, vex-2), sin-assign(rex, vex-add-int(vex-3, 1)))

Lifting Lingua-V to Lingua-FT (5)

syntax – formulas (metaconditions)

mec : MetCon-FT =

mec-make-mec(MetConVar-FT)		
mec-stronger(Con-FT , Con-FT)		in concrete syntax $\Rightarrow$
mec-weakly-equivalent(Con-FT , Con-FT)		in concrete syntax $\Leftrightarrow$
mec-less-defined(Con-FT , Con-FT)		in concrete syntax $\sqsubseteq$
mec-strongly-equivalent(Con-FT , Con-FT)		in concrete syntax $\equiv$
mec-insures LR(Con-FT , SpeIns-FT)		
mec-hereditary(Con-FT , MetPro-FT)		
mec-immunizing(Con-FT)		
mec-metaprogram(Con-FT , SpePro , Con-FT)		
...		
mec-and(MetCon-FT , MetCon-FT)		classical connectives
mec-or(MetCon-FT , MetCon-FT)		
mec-implies(MetCon-FT , MetCon-FT)		
mec-not(MetCon-FT		

Lifting Lingua-V to Lingua-FT (6)

syntax – examples of formulas (in colloquial syntax)

ground formulas in **Lingua-FT** i.e., metaformulas in **Lingua-V**:

$$\sqrt[2]{x} > 2 \Leftrightarrow x > 4$$

```
pre nni(x, k) and-k n+1 ≤ M:
  x := 0;
  while x+1 ≤ n do x := x+1 od
post x = n
```

free formulas in **Lingua-FT** (axioms):

```
pre sin @ con :
  sin
post con
```

```
pre prc-1: spr-1 post poc-1
pre prc-2: spr-2 post poc-2
poc-1 ⇒ prc-2
```

```
pre prc-1: spr-1; spr-2 post poc-2
pre prc-1: spr-1; asr poc-1 rsa; spr-2 post poc-2
pre prc-1: spr-1; asr prc-2 rsa; spr-2 post poc-2
```

(pre prc-1: spr-1 post poc-1) and pre ... post and pre ... post) **implies**
(pre ... post and pre ... post and pre ... post)

Lifting Lingua-V to Lingua-FT (7)

denotations – valuations

$\text{uni} : \text{Universe} = \text{Identifier} \mid \text{TypExpDen-V} \mid \text{RefExpDen-V} \mid \text{ValExpDen-V} \mid \dots$
 $\text{vlu} : \text{IndValuation} \subseteq \text{IndVar} \mapsto \text{Universe}$
 $\text{vlu} : \text{FunValuation} \subseteq \text{FunVar} \mapsto \{\text{fun} \mid \text{fun} : \text{Universe}^{c^*} \mapsto \text{Universe}\}$
 $\text{vlu} : \text{PreValuation} \subseteq \text{PreVar} \mapsto \{\text{pre} \mid \text{pre} : \text{Universe}^{c^*} \mapsto \text{Bool}\}$
 $\text{vlu} : \text{Valuation} \subseteq \text{IndValuation} \mid \text{FunValuation} \mid \text{PreValuation}$

Every valuation vlu is sort-wise well-formed, e.g.:

if $\text{vex} : \text{ValExpVar-FT}$ i.e., vex stands for an individual variable of sort
then $\text{vlu.vex} : \text{ValExpDen-V}$

e.g.,

$\text{vlu.vex-1} : \text{ValExpDen-V}$ here vex-1 is a concrete individual variable

Lifting Lingua-V to Lingua-FT (8)

denotations – examples of carriers

ved : ValExpDen-FT = Valuation $\mapsto$ ValExpDen-V
red : RefExpDen-FT = Valuation $\mapsto$ RefExpDen-V
ind : InsDen-FT = Valuation $\mapsto$ InsDen-V
spd : SpeProDen-FT = Valuation $\mapsto$ SpeProDen-V
ide : IdeDen-FT = Ide-FT
cod : ConDen-FT = Valuation $\mapsto$ ConDen-V
mcd : MetConDen-FT = Valuation $\mapsto$ MetConDen-V

Program construction rules (1st-order axioms)

1. Rules independent of **Lingua**:
 - a. formulas describing properties of metarelations
 - b. definitional formulas of behavioral metaconditions including the correctness of metaprograms,
2. Rules dependent of **Lingua**:
 - a. declaration-oriented rules,
 - b. rules concerning temporal metaconditions,
 - c. the rule on assignment instruction,
 - d. general implicative construction rules,
 - e. rules corresponding to structural instructions,

Program construction rules (1st-order axioms)

rules independent of **Lingua** – examples

Dependencies between metapredicates

$(\underline{\text{con1}} \equiv \underline{\text{con2}}) \text{ iff } ((\underline{\text{con1}} \sqsubseteq \underline{\text{con2}}) \text{ and } (\underline{\text{con2}} \sqsubseteq \underline{\text{con1}}))$
 $(\underline{\text{con1}} \Leftrightarrow \underline{\text{con2}}) \text{ iff } ((\underline{\text{con1}} \Rightarrow \underline{\text{con2}}) \text{ and } (\underline{\text{con2}} \Rightarrow \underline{\text{con1}}))$
 $(\underline{\text{con1}} \equiv \underline{\text{con2}}) \text{ implies } (\underline{\text{con1}} \Leftrightarrow \underline{\text{con2}})$
 $(\underline{\text{con1}} \equiv \underline{\text{con2}}) \text{ implies } (\underline{\text{con1}} \sqsubseteq \underline{\text{con2}})$
 $(\underline{\text{con1}} \Leftrightarrow \underline{\text{con2}}) \text{ implies } (\underline{\text{con1}} \Rightarrow \underline{\text{con2}})$

Relations $\equiv$ and $\Leftrightarrow$ are equivalences in the set of conditions

$\underline{\text{con}} \equiv \underline{\text{con}}$ reflexivity
 $(\underline{\text{con1}} \equiv \underline{\text{con2}}) \text{ implies } (\underline{\text{con2}} \equiv \underline{\text{con1}})$ symmetry

...

De Morgan's laws

$\text{not } (\underline{\text{con1}} \text{ and-k } \underline{\text{con2}}) \equiv (\text{not}(\underline{\text{con2}}) \text{ or-k not}(\underline{\text{con1}}))$
 $\text{not } (\underline{\text{con1}} \text{ and-k } \underline{\text{con2}}) \Leftrightarrow (\text{not}(\underline{\text{con2}}) \text{ or-k not}(\underline{\text{con1}}))$

...

Program construction rules (1st-order axioms)

rules dependent of **Lingua-V** - examples (1)

Rule for variable declaration

```
pre (ide is free) and-k (tex is type)
  let ide be tex tel
post var ide is tex
```

Rule for adding an abstract attribute

```
pre (ide-1 is free) and-k (ide-2 is class) and-k (tex is type) :
  let ide-1 be tex with yex as pst tel in ide-2
post att ide-1 is tex with yex in ide-2 as pst
```

Rule for adding a type constant

```
pre (ide-1 is free) and-k (ide-2 is class) and-k (tex is type) :
  set ide-1 be tex tes in ide-2
post ide-1 is tex
```

Program construction rules (1st-order axioms)

rules dependent of **Lingua-V** - examples (2)

Rules about temporal metaconditions

not(ide **is free**) **hereditary in** mpr
(ide is free) **co hereditary in** mpr
(ide is tex) hereditary in mpr

Rule for assignment instruction

pre sin **@** con
 sin
post con

Rule for conditional branching

pre (prc **and-k** vex) : sin1 **post** poc
pre (prc **and-k** (**not-k** vex)) : sin2 **post** poc
prc $\Rightarrow$ (vex **or-k** (**not-k** vex))

pre prc : **if** vex **then** sin1 **else** sin2 **fi** **post**
poc

Denotational models of ecosystems: **Lingua-E**

An ecosystem as a language with denotational semantics

Introductory domains

car : Character = {a, b, c, ..., A, B, C, ..., 0, 1, ..., 9, (,), ...}
mcn : MetConNam = Character^{c+} the names of metaconditions
rep : Repository = MetConNam $\Rightarrow$ ValMetCon-FT

The domains of the algebra of denotations of **Lingua-E**:

tex : Text = Character^{c+}
mcn : MetConNam = Text
sud : SubDen = IndVal-FT $\Rightarrow$ Text the denotations of substitution
acd : ActDen = Repository $\mapsto$ Repository | Error the denotations of actions
inv : IndVar-FT = IdeVar-FT | ValExpVar-FT | RefExpVar-FT... individual var.
ide : IdeVar-FT = ide | {ide-tex | tex : Text} ,ide' – variables with indices
vex : ValExpVar-FT = vex | {vex-tex | tex : Text}
rex : RefExpVar-FT = rex | {rex-tex | tex : Text}
sin : SpeInsVar-FT = sin | {sin-tex | tex : Text}
con : ConVar-FT = con | prc | poc |
 | {con-tex | tex : Text} |
 | {prc-tex | tex : Text} |
 | {poc-tex | tex : Text}

Categories of actions:

- standard actions,
- specific actions,
- logistic actions,
- proving actions.

The meanings of inference rules in the context of our ecosystem

Rule of substitution

$\vdash \text{mec}$
 inv is free in mec
 ter is of sort of inv

 $\vdash \text{mec}[\text{inv}/\text{ter}]$

If
 mec is in repository **and...**
then
 $\text{mec}[\text{inv}/\text{ter}]$ may be stored in repository

Rule of detachment

$\vdash \text{mec1}$
 $\vdash (\text{mec1} \text{ **implies** } \text{mec2})$

 $\vdash \text{mec2}$

If
 mec1 is in repository
and
 $(\text{mec1} \text{ **implies** } \text{mec2})$ is in repository
then
 mec2 may be stored in repository

We need corresponding actions to implement both rules

These rules are not formulas in **Lingua-FT**

mec , inv , ter ,... are variables in a **MetaLingua-FT**

Denotational models of ecosystems

Constructors of substitution denotations

Auxiliary functions:

$\text{free} : \text{IndVar-FT} \times \text{MetCon-FT} \mapsto \{\text{tt}, \text{ff}\}$ variable is free in metacondition
 $\text{matching} : \text{IndVar-FT} \times \text{Text} \mapsto \{\text{'OK'}\} \mid \text{Error}$ the sort of a variable matches the sort of text

Constructors of the denotations of substitutions:

$\text{create-sub} : \text{IndVar-FT} \times \text{Text} \mapsto \text{SubDen}$

$\text{create-sub}(\text{inv}, \text{ter}) =$

not matching(inv, tex) $\rightarrow$ 'matching not satisfied'

true $\rightarrow$ [inv/ter]

$\text{expand-sub} : \text{SubDen} \times \text{IndVar-FT} \times \text{Text} \mapsto \text{SubDen}$

$\text{expand-sub}(\text{sub}, \text{inv}, \text{tex}) =$

$\text{sub.inv} = !$ $\rightarrow$ 'variable already assigned'

not matching(inv, tex) $\rightarrow$ 'matching not satisfied'

true $\rightarrow$ sub[inv/ter]

Denotational models of ecosystems

Constructor of the denotation of a substitution action

Auxiliary functions:

$\text{swap} : \text{IndVar-FT} \times \text{Text} \mapsto \text{MetCon-FT} \mapsto \text{MetCon-FT} \mid \text{Error}$ single swap
 $\text{replace} : \text{SubDen} \mapsto \text{MetCon-FT} \mapsto \text{MetCon-FT} \mid \text{Error}$ many swaps

$\text{substitute} : \text{MetConNam} \times \text{SubDen} \times \text{MetConNam} \mapsto \text{ActDen}$ i.e.
 $\text{substitute} : \text{MetConNam} \times \text{SubDen} \times \text{MetConNam} \mapsto \text{Repository} \mapsto \text{Repository} \mid \text{Error}$
 $\text{sub-gro.}(\text{mcn-s}, \text{sud}, \text{mcn-t}).\text{rep} =$ -s – source, -t – target

$\text{rep.mcn-s} = ? \rightarrow \text{'no source metacondition'}$
 $\text{rep.mcn-t} = ! \rightarrow \text{'target name already assigned'}$

let

$\text{mec-s} = \text{rep.mcn-s}$

$\text{mec-t} = \text{replace.sud.so-mec}$

$\text{mec-t} : \text{Error} \rightarrow \text{mec-t}$

true $\rightarrow \text{rep}[\text{mcn-t}/\text{mec-t}]$

Rule of substitution

$\mid = \text{mec}$
 inv is free in mec
 ter is of sort of inv

 $\mid = \text{mec}[\text{inv}/\text{ter}]$

Due to the rule of substitution,
constructor substitute is sound.

Denotational models of ecosystems

Standard actions – detachment

Rule of detachment

$\frac{\begin{array}{l} \models \text{mec1} \\ \models (\text{mec1} \textbf{ implies } \text{mec2}) \end{array}}{\models \text{mec2}}$

$\text{detach} : \text{MetConNam} \times \text{MetConNam} \times \text{MetConNam} \mapsto \text{ActDen}$

$\text{detach} : \text{MetConNam} \times \text{MetConNam} \times \text{MetConNam} \mapsto \text{Repository}$

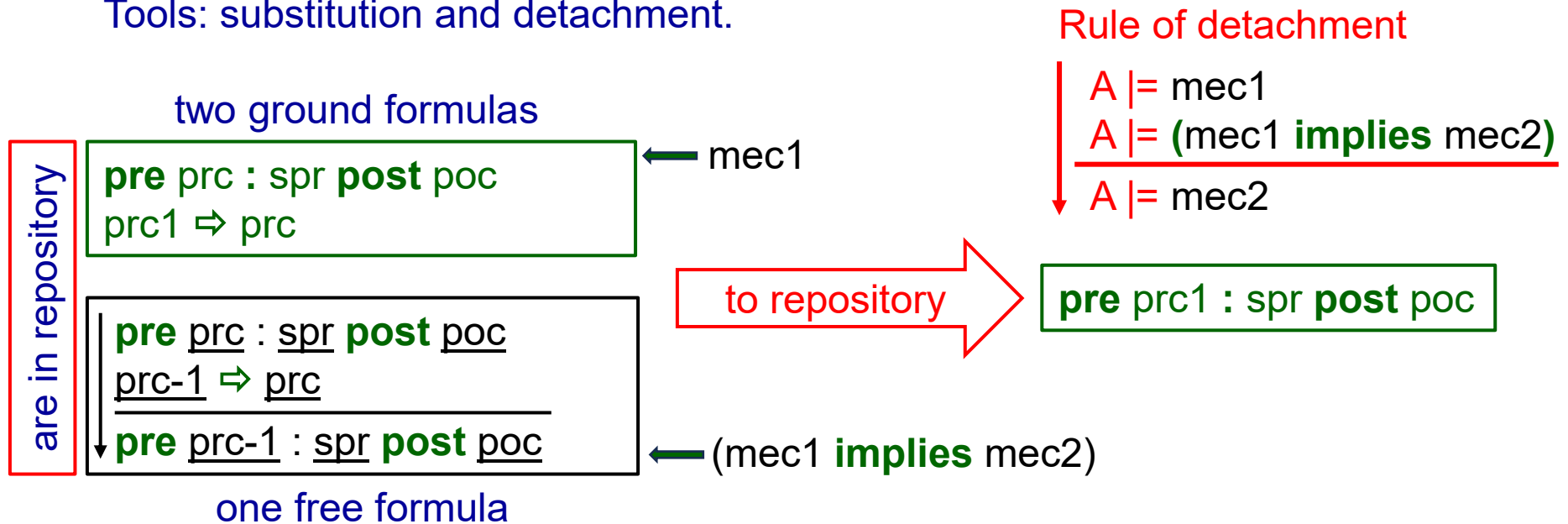
$\mapsto \text{Repository} \mid \text{Error}$

Due to the rule of detachment,
constructor `detach` is sound.

Are substitution- and detachment-action adequate candidates for everyday tools (1)?

A programmer's task: given two metaconditions and a rule generate a new metacondition.

Tools: substitution and detachment.



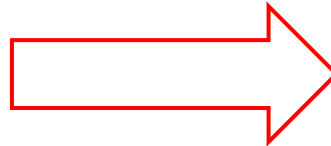
Steps:

1. define and apply to the construction rule a substitution action to get
 $((\text{pre } \underline{\text{prc}} : \text{spr } \text{post } \underline{\text{poc}}) \text{ and } (\text{prc1} \Rightarrow \text{prc})) \text{ implies } (\text{pre } \text{prc1} : \text{spr } \text{post } \text{poc})$
2. to be able to apply detachment generate and store (combine two formulas)
 $((\text{pre } \underline{\text{prc}} : \text{spr } \text{post } \underline{\text{poc}}) \text{ and } (\text{prc1} \Rightarrow \text{prc}))$
3. apply detachment to 1. and 2.

Are substitution- and detachment-action adequate candidates for everyday tools (2)?

A subtask of our programmer: step 2

pre prc : spr **post** poc
prc1 $\Rightarrow$ prc



(**pre** prc1 : spr **post** poc)
and
(prc1 $\Rightarrow$ prc)

Steps:

1. identify in repository an axiom (a free formula):
(mec1 **implies** (mec2 **implies** (mec1 **and** mec2)))
2. substitute appropriately to get
((**pre** prc : spr **post** poc) **implies** ((prc1 $\Rightarrow$ prc) **implies** ((**pre** prc : spr **post** poc) **and** (prc1 $\Rightarrow$ prc))))
3. detach to get:
((prc1 $\Rightarrow$ prc) **implies** ((**pre** prc : spr **post** poc) **and** (prc1 $\Rightarrow$ prc)))
4. detach again to get
((**pre** prc : spr **post** poc) **and** (prc1 $\Rightarrow$ prc)))

7 steps to perform a **single** replacement of a precondition by a stronger one

Two derived inference rules

If the following inference rule is sound

$$\frac{\begin{array}{l} \models \text{mec1} \\ \models \text{mec2} \end{array}}{\models \text{mec1} \text{ and } \text{mec2}} \quad \begin{array}{l} \text{due to} \\ \text{lemma in} \\ \text{repository} \end{array} \quad \models (\text{mec1} \text{ implies } (\text{mec2} \text{ implies } (\text{mec1} \text{ and } \text{mec2})))$$

Similarly, the following inference rule is sound

$$\frac{\begin{array}{l} \models \text{pre } \text{prc} : \text{spr } \text{post } \text{poc} \\ \models \text{prc1} \Rightarrow \text{prc} \end{array}}{\models \text{pre } \text{prc1} : \text{spr } \text{post } \text{poc}} \quad \begin{array}{l} \text{due to} \\ \text{lemma in} \\ \text{repository} \end{array} \quad \frac{\begin{array}{l} \text{pre } \underline{\text{prc}} : \underline{\text{spr}} \text{ post } \underline{\text{poc}} \\ \underline{\text{prc1}} \Rightarrow \underline{\text{prc}} \end{array}}{\text{pre } \underline{\text{prc1}} : \underline{\text{spr}} \text{ post } \underline{\text{poc}}}$$

$((\text{pre } \underline{\text{prc}} : \underline{\text{spr}} \text{ post } \underline{\text{poc}}) \text{ and } (\underline{\text{prc1}} \Rightarrow \underline{\text{prc}})) \text{ implies } (\text{pre } \underline{\text{prc1}} : \underline{\text{spr}} \text{ post } \underline{\text{poc}})$

strengthen-pre : MetConNam x MetConNam x MetConNam $\mapsto$ Repository
 $\mapsto$ Repository | Error

pre prc : spr post poc

prc1 $\Rightarrow$ prc

pre prc1 : spr post poc

The sound action of strengthening precondition

strengthen-pre : MetConNam x MetConNam x MetConNam $\mapsto$ Action
strengthen-pre : MetConNam x MetConNam x MetConNam $\mapsto$ Repository
 $\mapsto$ Repository | Error

strengthen-pre.(mcn-m, mcn-s, mcn-t).rep = -m – **metapr.**, -s – **stronger**, -t – **target**

rep.mcn-m = ? $\rightarrow$ 'no prerequisite metaprogram'
rep.mcn-s = ? $\rightarrow$ 'no stronger-than metacondition'
rep.mcn-t = ! $\rightarrow$ 'target name already assigned'
not is-metaprogram.(rep.mcn-m) $\rightarrow$ 'metaprogram expected'
root.(rep.mcn-s) $\neq$ **stronger** $\rightarrow$ 'a stronger-than metacondition expected'

let

pre prc : spr **post** poc = rep.mcn-m
 stronger(prc1, con) = rep.mcn-s

con $\neq$ prc $\rightarrow$ 'conclusion not adequate'

let

 new-mec = **pre** prc1 : spr **post** poc

true $\rightarrow$ rep[mcn-t/new-mec]

Note! No substitution, no detachment.

Coming back to our programmer's task

Two ground formulas in repository

my-prg $\rightarrow$ **pre** prc : spr **post** poc
my-mec $\rightarrow$ prc1 $\Rightarrow$ prc

to repository

new-prg $\rightarrow$ **pre** prc1 : spr **post** poc

Activate:

strengthen-pre(my-prg, my-mec, new-prg)

Denotational models of ecosystems

specific actions – the removal of an assertion

A new nonderivable inference rule

The removal of an assertion

$$\frac{\downarrow \models \text{pre prc : head ; asr con rsa ; tail post poc}}{\downarrow \models \text{pre prc : head ; tail post poc}}$$

This rule **can't be derived** from a formula in **Lingua-FT**, since scripts

pre prc : head ; **asr** con **rsa** ; tail **post** poc

pre prc : head ; tail **post** poc

are not in MetCon-FT.

They are not metaconditions!

We have to add this rule as an extension of our logic.

Denotational models of ecosystems

specific actions – Lingua-V-oriented

A new nonderivable inference rule

The replacement of an assertion

```
|= pre prc : head ; asr con rsa ; tail post poc  
|= con ⇔ con1  
—  
↓ |= pre prc : head ; asr con1 rsa ; tail post poc
```

More such cases:

- the replacement of a boolean value-expression in a program by a strongly equivalent expression,
- the introduction of an assertion block into a program,
- adding a register identifier to a program.

Denotational models of ecosystems

prover-activation action

$\text{prove} : \text{MetCon-FT} \times \text{MetConNam} \mapsto \text{Repository} \rightarrow \text{Repository} \mid \text{Error}$

$\text{prove}.\text{(mec, mcn).rep} =$

$\text{rep.mnc} = ! \quad \rightarrow \text{'name already assigned'}$

$\text{valid.mec} : \text{Error} \quad \rightarrow \text{valid.mec}$

$\text{valid.mec} = ? \quad \rightarrow ?$

$\text{valid.mec} = \text{'NO'} \quad \rightarrow \text{'metacondition not valid'}$

$\text{true} \quad \rightarrow \text{rep[mcn/mec]}$

in this definition:

$\text{valid} : \text{MetCon-FT} \rightarrow \{\text{YES, NO}\} \mid \text{Error} \quad \text{a partial function!}$



Thank you for
your attention

